Appendix

To be more precise about the quantities we discussed, let's suppose we have a portfolio "x" where weight w_1^x is invested in the first security in the universe of investable securities, w_2^x in the second investable security… and so forth up to w_n^x in the n^{th} and final security in the universe of investable securities. These weights add to 1 (100%)—mathematically that is the budget constraint $\sum_{i=1}^{n} w_i^x = 1.$

In the standard fixed-income framework, we are able to list all planned future cash flows of an instrument and discount them to the present with an appropriate default-free discount curve. The standard framework further assumes that we can add a spread to the discount rates in order to match the market price of the instrument; this spread accounts for credit risk, liquidity and other factors. The sensitivity of the instrument's price to changes in this spread is called the spread duration.

We will assume that spread durations are option-adjusted: for example, if a bond has a call schedule that gives the issuer the option to pay off the bond before its maturity date, then there is optionality associated with the bond. The calculations of spread and spread duration need to take this optionality into account. Sometimes the option adjustment is done by generating multiple random (Monte Carlo) paths into the future; in some of those paths the options will be in the money, and in others the options will be out of the money. The option adjustment would average the behavior across the multiple paths.

The resulting spreads and spread durations are called option-adjusted and are abbreviated OAS and OASD, respectively. If there is no optionality in the instrument, then option-adjusted spread duration is just the same as spread duration. We will assume that everything is option-adjusted, so if we refer just to "spread" or "spread duration," we mean "option-adjusted spread" and "option-adjusted spread duration," respectively.

Security i's duration-times-spread DTS_i is simply $DTS_i = SD_i OAS_i$. The duration-times-spread of portfolio x, $DTS_x = \sum_{i=1}^{n} w_i^x DTS_i$, i.e. it is the weighted sum of the DTS's of the individual securities in the portfolio.

The (unadjusted) spread duration of portfolio x is $SD_x = \sum_{i=1}^{n} w_i^x SD_i$, and the option-adjusted spread of portfolio x is $OAS_x = \sum_{i=1}^{n} w_i^x OAS_i$.

Adjusted spread duration assumes there is an index or benchmark that is appropriate for adjustments. We define $ASD_i = SD_i \left(\frac{OAS_i}{OAS_b}\right) = \left(\frac{DTS_i}{OAS_b}\right)$, where b is the appropriate benchmark. This means that the adjustment to spread duration might differ by context if the security is in two different portfolios with two different benchmarks.

The adjusted spread duration of a portfolio is $ASD_x = \sum_{i=1}^{n} w_i^x ASD_i = \frac{1}{OAS_b} \sum_{i=1}^{n} w_i^x DTS_i = \frac{DTS_x}{OAS_b}$. In other words we can get an adjusted spread duration by adding up the weighted adjusted spread durations of each security in the portfolio, or we can just divide the DTS of the portfolio by the OAS of its benchmark. Note that the adjusted spread duration of the benchmark is NOT the spread duration of the benchmark, i.e. $ASD_b = \frac{DTS_b}{OAS_b} \neq \sum_{i=1}^{n} w_i^b SD_i$. The use of ratios makes the adjusted spread duration metric less vulnerable to the current market environment.

Contributions to adjusted spread durations can also be computed for segments of portfolios and subtracted to get benchmark-relative exposures of segments. Contributions to adjusted spread duration are better than contributions to DTS, again because of the avoidance of procyclicality.

GARCH(1,1) models estimate three parameters $\alpha, \beta, and\ \gamma$ so that $\sigma_t^2 = \gamma + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2$ is a good predictor of the volatility of the time series y_t. Generally we would like to see that the de-GARCHED time series y_t / σ_{t-1} moves in the direction of white noise, with low kurtosis and a good Ljung-Box test statistic.